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Finding the fixed point of a function using the Bisection-Ant Lion Optimizer Algorithm

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ABSTRACT

In this paper, we introduce a new iterative method to finding the fixed point of a nonlinear function. Therefore, we merge ideas proposed in Ant Lion Optimizer Algorithm and Bisection method . This method is new and very efficient for solving a nonlinear equation. We explain this method with three benchmark functions and compare results with others methods, such as ALO, MVO, SSA .

KEYWORDS: meta-heuristic algorithms , Fixed point problems, Ant Lion Optimizer Algorithm , Bisection algorithm.

1 INTRODUCTION

Obtaining the roots of equations, especially nonlinear equations, is one of the most important topics in engineering and basic sciences. For this sake, many researchers have checked this problem for some years [34,42].

The Bisection method is one of the most important methods in numerical calculations to find the root of a continuous function, which we know has a different sign at two points. This method is one of the simplest ways to find the root of a function in numerical calculations.

Meta-heuristic or meta-heuristic or meta-heuristic algorithms are a type of random algorithms that are used to find the optimal answer. Optimization methods and algorithms are divided into two categories: exact algorithms and approximate algorithms.

Well-known population- based meta-heuristic algorithms include evolutionary algorithms (genetic algorithm) [2], ant colony optimization (ACO) [3, 4], bee colony(BC) [5], particle swarm optimization method (PSO) [6], forest optimization algorithm (FO) [7], Battle royale optimization algorithm (BRO) [8], runner- root algorithm(RRA) [9] , intelligent water drops algorithm (IWD)

[10], Artificial Bee Colony algorithm(ABC) [11, 12], Firefly Algorithm(FA) [13] , Differential evolution (DE) algorithms [14], biogeography based optimization (BBO) algorithm [15].

In recent years, new meta-heuristic algorithms have been developed with respect to living organisms in nature (inspired by nature), the most famous of which are the Gray Wolf Optimization Algorithm(GWO) [16], the Dragonfly algorithm (DA) [17], the Flower Pollination Optimization Algorithm (FPA) [18], Whale optimization Algorithm (WOA) [19], Grasshopper Optimisation Algorithm (GOA) [20], social spider algorithm (SSA) [21], Sine Cosine Algorithm (SCA) [22], Multi-Verse Optimizer algorithm (MVO) [23], Moth-flame optimization algorithm (MFO) [24], Ant Lion Optimizer algorithm (ALO) [25], Emperor Penguins Colony algorithm [26] and so on [1,44], [27-33].

In this paper,we introduce a novel iterative method that combines the advantages of both the Bisection method and the Ant Lion Optimizer Algorithm to solve the hard fixed point problem.

In Sect. 2, the Bisection method and the Ant Lion Optimizer Algorithm is explained and fixed point problem is illustrated. Also suggested method illustrated in Sect.3. Section 4 measures the resolution of the offered method by different methods on several functions. Also, the result is available at Sect. 5.

2 PRELIMINARIES

In the present section, the Bisection method and the Ant Lion Optimizer Algorithm is explained and fixed point problem is illustrated.

2.1 The Ant Lion Optimizer Algorithm

The Antlion Optimizer, called ALO for short, based on the behavior of ants in the wild, was first introduced in 2015. Numerous optimization methods have been proposed so far, but many of them have encountered problems with a variety of optimization functions. However, in the face of 23 optimization functions, the ant lion optimization algorithm simply avoids local optimizations and converges to the global optimization.

Antlions (doodlebugs) belong to the Myrmeleontidae family and Neuroptera order (net-winged insects). The lifecycle of antlions includes two main phases: larvae and adult. A natural total lifespan can take up to 3 years, which mostly occurs in larvae (only 3–5 weeks for adulthood). Antlions undergo metamorphosis in a cocoon to become adult. They mostly hunt in larvae and the adulthood period is for reproduction.Their names originate from their unique hunting behaviour and their favourite prey. An antlion larvae digs a cone-shaped pit in sand by moving along a circular path and throwing out sands with its massive jaw. The edge of the cone is sharp enough for insects to fall to the bottom of the trap easily. Once the antlion realizes that a prey is in the trap, it tries to catch it. However, insects usually are not caught immediately and try to escape from the trap. In this case, antlions intelligently throw sands towards to edge of the pit to slide the prey into the bottom of the pit. When a prey is caught into the jaw, it is pulled under the soil and consumed. After consuming the prey, antlions throw the leftovers outside the pit and amend the pit for the next hunt. Another interesting behaviour that has been observed in life style of antlions is the relevancy of the size of the trap and two things: level of hunger and shape of the moon. Antlions tend to dig out larger traps as they become more hungry and/or when the moon is full .They have been evolved and adapted this way to improve their chance of survival. It also has been discovered that an antlion does not directly observe the shape of the moon to decide about the size of the trap, but it has an internal lunar clock to make such decisions .

The main inspiration of the ALO algorithm comes from the foraging behaviour of antlion's larvae. In the next subsection the behaviour of antlions and their prey in nature is first modelled mathematically. An optimization algorithm is then proposed based on the mathematical model.

The ALO algorithm mimics interaction between antlions and ants in the trap. To model such interactions, ants are required to move over the search space, and antlions are allowed to hunt them and become fitter using traps. Since ants move stochastically in nature when searching for food, a random walk is chosen for modelling ants' movement. During optimization, the following conditions are applied:

- Ants move around the search space using different randomwalks.
- Random walks are applied to all the dimension of ants.
- Random walks are affected by the traps of antlions.
- Antlions can build pits proportional to their fitness (the higher fitness, the larger pit).
- Antlions with larger pits have the higher probability to catch ants.

- Each ant can be caught by an antlion in each iteration and the elite (fittest antlion).

- The range of random walk is decreased adaptively to simulate sliding ants towards antlions.

- If an ant becomes fitter than an antlion, this means that it is caught and pulled under the sand by the antlion.

- An antlion repositions itself to the latest caught prey and builds a pit to improve its change of catching another prey after each hunt.

Figure 1 and Figure 2 provide the above explanations for further understanding.

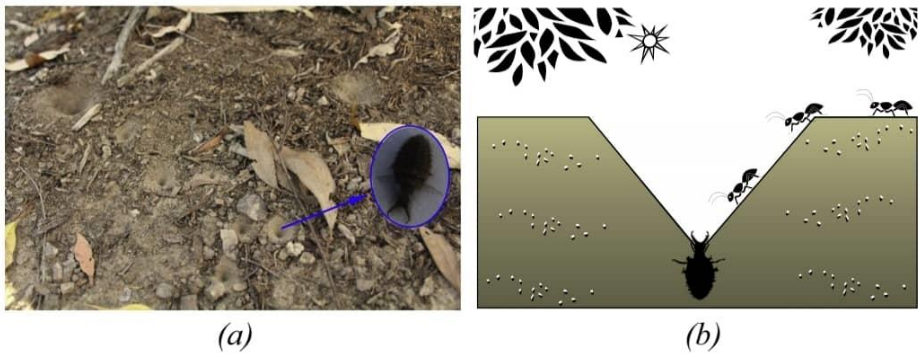


Fig. 1. Cone-shaped traps and hunting behaviour of antlions.

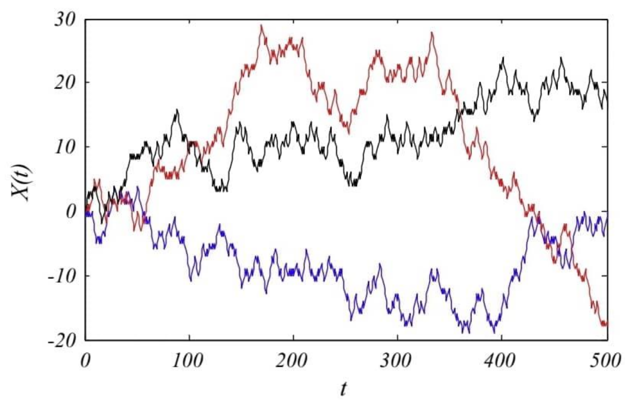


Fig. 2. Three random walks.

2.2 Definition the fixed point

In mathematics, a fixed point (invariant point) of a function isa point that is mapped to itself by the function. In other word ,a number c is a fixed point for a given function g if g(c) = c. A set of fixed points is sometimes called a fixed set. An iterative method for solving equation g(x) = x is the recursive relation xi+1= g(xi), i = 0, 1, 2, . . ., with some initial guess x0. The algorithm stops when one of the following stopping criterion is met:

•D1: total number of iterations is N, for some N, fixed a priori.

•D2: |xi+1− xi| < € for some €, fixed a priori.

This procedure is shown in figure 3.

Repeat

1. Give the transcendental equation $x = g(x)$,
2. Give initial guess in interval $[a, \ b]$,
3. Do $x_{i+1} = g(x_i)$

until (D_1 or D_2 is true)

Figure 3: Fixed Point iteration scheme.

2.3 Definition of the Bisection method

The Bisection method is a numerical method for estimating the roots of a real-valued function. Given a continuous function f on an interval [a, b], where f(a) and f(b) have opposite signs, the problem is to find x that satisfies f(x) = 0. Figure 4 gives bisection method to compute the roots of a function.

Input: A continuous function $f(x)$, interval [a,b], tolerance: TOL; maximum number of iterations: N_0

1. If $(b-a)/2 < TOL$ then exit
2. Set $i = 1$
3. while $i \leq N_o$ and $(b-a)/2 \geq TOL$ do
   Set $c = a + (b-a)/2$
   Compute $f(c)$
   If $f(c) = 0$ then output c and exit
   If $f(a).f(c) > 0$ then set $a = c$ and $f(a) = f(c)$
   else set $b = c$ and $f(b) = f(c)$
   Set $i = i + 1$.
4. If $(b-a)/2 < TOL$ then output c, 'tolerance limit exceeded '
5. If $i > N_o$ then output c, 'number of iterations reached N_o'

Figure 4: Bisection method to compute the roots of a function

3 THE BISECTION- ANT LION OPTIMIZER ALGORITHM

At present part, we present one modern repetitious procedure with incorporating the Bisection method and the ALO algorithm (BALO) to gain the solution estimation of a fixed point question as g(x) = x. We describe a function f(x) = g(x) - x. Accordingly the question of discovering the fixed points of g(x) is decreased to discovering the roots of f(x). We subsequent describe a function h(x) =|f(x)|. The question of discovering the roots of f(x) is better decreased to discovering an x that minimizes h(x). The opinion where is that for obtaining the half point of the distance I to begin with one volunteer answer, ALO algorithm is utilized to impute one superior estimation and determined a distance Ik = [ak,bk] one volunteer solution xk is calculated utilizing the ALO algorithm. If f(xk) = 0 we are accomplished, again calculate one modern distance Ik+1 into Ik pertaining against whether f(xk).f (ak) < 0 or f(xk).f (bk) < 0.

4 IMPLEMENT METHODS ON VARIOUS FUNCTIONS

In this section, we illustrate our algorithm with some examaples and compare the results with other evolutionary optimization algorithms such as ALO, MVO, SSA .

4.1 Introducing different functions

Introducing different functions

$$g_1(x): \frac{x^3}{150} - 2\sin(x) = x \quad ; \quad x \in [-10, +10] \quad ; \quad fp = 0$$

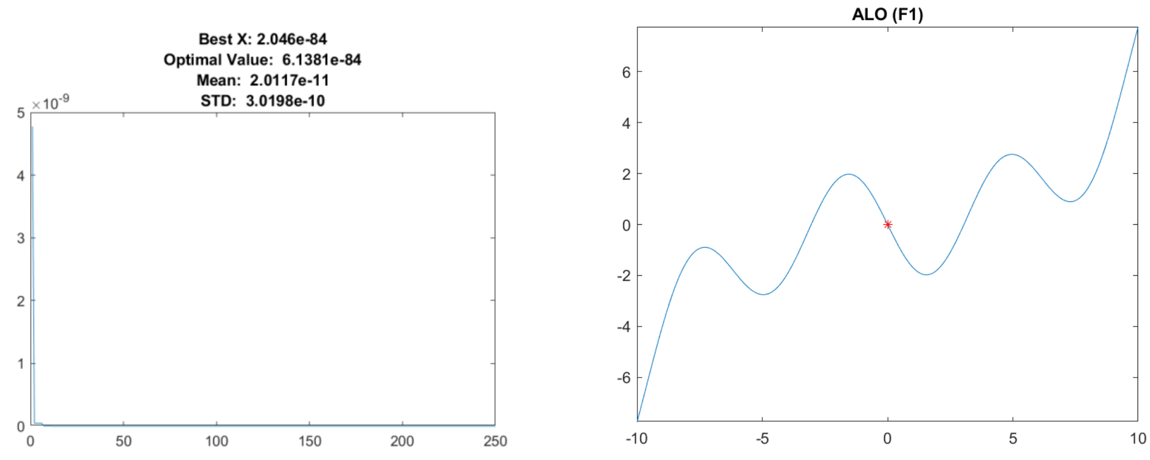
$$g_2(x): 3x^4 + 4\cos(x) - 4 = x \quad ; \quad x \in [-15, +15] \quad ; \quad fp = 0$$

$$g_3(x): x cos(x) - \frac{x^2}{3}\sin(x) + 2\pi = x \quad ; \quad x \in [-2\pi, +2\pi] \quad ; \quad fp = \pi$$

Results for the three functions are shown in Table 1 and Their diagrams are also shown in Figures 5-7. Figures 5 to 7 show Diagram of the recovery process of the g1 to g3 functions by the BALO algorithm in (a) and diagram of the finding of the fixed point of the g1 to g3 functions by the BALO algorithm in (b).

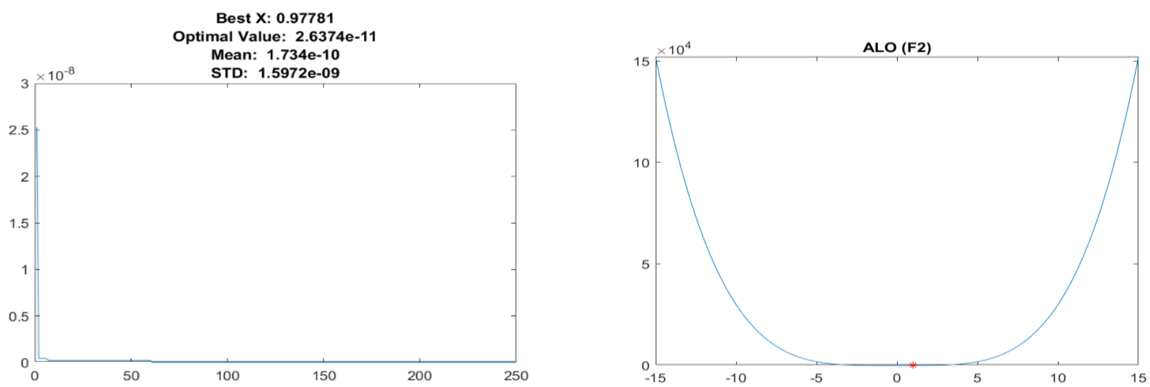
Table 1 The Comparative results obtained for each function by ALO, MVO, SSA and BALO algorithms

algorithm	Components	g1(x)	g2(x)	g3(x)
ALO	error	3.42E-10	1.35E-09	9.76E-12
	X_best	3.141593	-1.3E-09	-3.3E-12
	mean(e)	6.73E-06	4.65E-05	3.58E-05
	std(e)	2.75E-05	0.000172	0.000547
MVO	error	1.78E-08	2.19E-07	2.48E-08
	X_best	3.141593	-2.2E-07	-8.3E-09
	mean(e)	8.25E-05	0.000333	0.000101
	std(e)	0.000394	0.002159	0.001331
SSA	error	3.79E-11	4.99E-10	4.46E-10
	X_best	3.141593	0.97781	-1.5E-10
	mean(e)	2.75E-05	5.77E-05	3.32E-05
	std(e)	0.000126	0.000718	0.000134
BALO	error	0	1.27E-11	1.11E-89
	X_best	3.141593	0.97781	-3.7E-90
	mean(e)	3.91E-11	1.73E-10	2.01E-11
	std(e)	1.71E-10	1.6E-09	3.02E-10



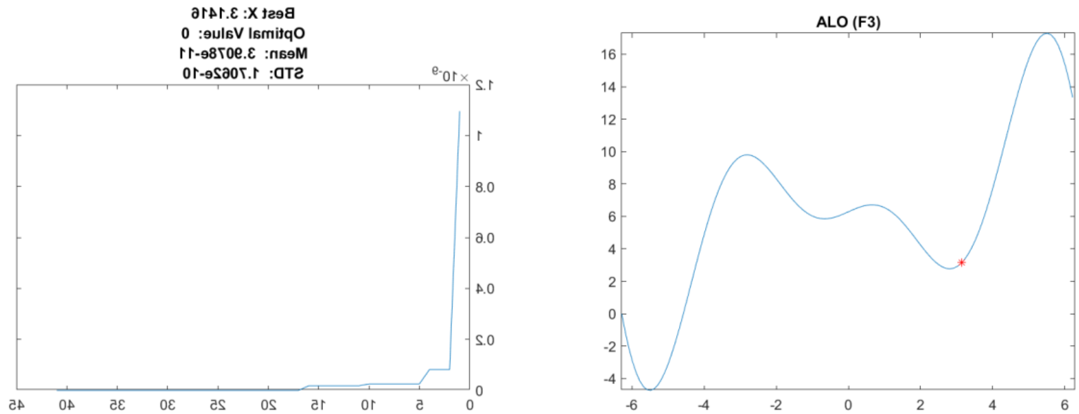
(a) (b)

Figure ***5***: Diagram of the recovery process of the g1 function by the BALO algorithm in (a) and diagram of the finding of the fixed point of the g1 function by the BALO algorithm using the intersection of the diagram g1 (x) = x in (b)



(a) (b)

Figure ***6***: Diagram of the recovery process of the g2 function by the BALO algorithm in (a) and diagram of the finding of the fixed point of the g2 function by the BALO algorithm using the intersection of the diagram g2 (x) = x in (b)



(a) (b)

Figure *7*: Diagram of the recovery process of the g3 function by the BALO algorithm in (a) and diagram of the finding of the fixed point of the g3 function by the BALO algorithm using the intersection of the diagram g3 (x) = x in (b)

5 CONCLUSION

In this paper, we introduce a novel iterative method for finding a fixed point of a function g in a real interval [a, b] ⊆ R by using Ant Lion Optimizer Algorithm and Bisection method. If the function g is hard, it is sometimes difficult to determine suitable initial value close to the location of a fixed point. Derivative method (find the derivative of g(x) − x and find its root) is also sometimes not useful for various reasons like the derivative may not exist, the derivative is hard to compute or finding the root of a derivative itself may be difficult. ALO algorithm helps in finding a good initial value and the proposed method does away with the need to compute the derivative. Our proposed algorithm is easy to use and reliable. As comparison with other algorithm shows, the accuracy of our proposed method also is good.

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